Real Analysis II
Revision

Definitions

Definition 1. Topology: open and closed sets, bounded, limit points, closure, compact, Heine-Borel theorem, Cantor set.
Continuity: continuity (all definitions), uniform continuity, classification of discontinuities, intermediate value property.
Sequences and series: pointwise convergence, uniform convergence, power series, Taylor series, Maclaurin series.
Integration: upper and lower sums, upper and lower integrals, Riemann integral, conditions for Riemann-integrability.

Theorems

Theorem. *The following theorems should be known, without proofs:*

1. *sequential criterion for functional limits;*
2. *sequential criterion for discontinuity;*
3. *divergence criterion for functional limits;*
4. *extreme value theorem;*
5. *sequential criterion for non uniform continuity;*
6. *intermediate value theorem;*

   You won't be asked to *give* the proof but you may be asked *about* the proof, so you should know it.
7. *Darboux's theorem;*
8. *generalised mean value theorem;*
9. *l'Hôpital's Theorem;*
10. *a function continuous on a compact set is uniformly continuous on that set;*
11. *Cauchy criterion for uniform convergence of a family of functions (**Since this is studied in Real Analysis I the proof may be tested as an unseen question**);*
12. *the so called **interchange theorem** of sequences and differentiation: for a pointwise convergent family $(f_n) \to f$, where each f_n is differentiable, if $(f_n') \to g$ converges uniformly then $f' = g$;*
13. *Weierstrass M-test;*
14. *Lagrange's remainder theorem;*
15. *Lebesgue-Vitali characterisation of Riemann-integrability.*

Theorem. *The following theorems should be known, both statements and proofs:*

1. *nested interval property for compact sets;*
2. *Cantor's set is closed;*

3. *preservation of compact sets;*
4. *derivatives do not have jump discontinuities;*
5. *differentiable functions are continuous;*
6. *interior extremum theorem;*
7. *Rolle's theorem;*
8. *mean value theorem;*
9. *Dirichlet's function is nowhere continuous;*
10. *Dirichlet's modified function is continuous at one point;*
11. *Thomae's function is discontinuous at rational numbers.*

Continuity

Question 1. Classify discontinuities of a given function.

Question 2. Decide whether a given function is (uniformly) continuous or not. Prove a given function is (uniformly) continuous.

Derivatives

Question 3. Compare the strength of continuity vs differentiability.

Question 4. Proofs and applications of main theorems.

Question 5. Study a continuous, nowhere differentiable function.

Sequences and series

Question 6. Determine conditions for when, given a family of functions, the limit function is continuous.

Question 7. Calculation of Taylor series of a given function.

Question 8. Application of Weierstrass M-test.

Question 9. Use Lagrange's remainder theorem to estimate the error of power series approximation.

Integration

Question 10. Determine whether a given function is Riemann-integrable (using upper and lower integrals).